Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Friday 17 May 2024

Afternoon

Paper reference **8FM0/26**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1.

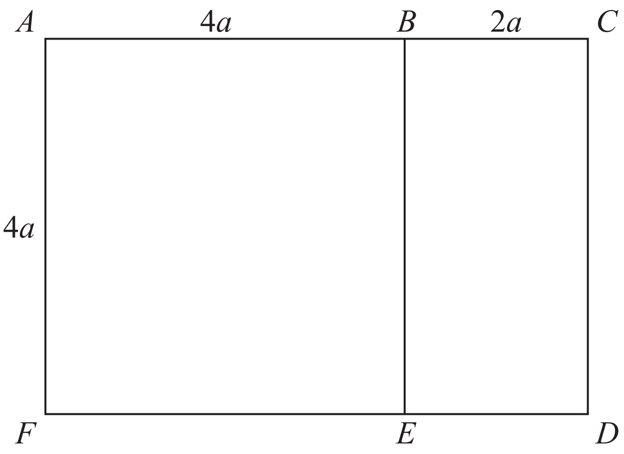

Figure 1

A uniform rod of length $24a$ is cut into seven pieces which are used to form the framework *ABCDEF* shown in Figure 1.

It is given that

- $AF = BE = CD = AB = FE = 4a$
- $BC = ED = 2a$
- the rods *AF*, *BE* and *CD* are parallel
- the rods *AB*, *BC*, *FE* and *ED* are parallel
- *AF* is perpendicular to *AB*
- the rods all lie in the same plane

The distance of the centre of mass of the framework from *AF* is d.

(a) Show that $d = \dfrac{19}{6}a$ **(4)**

(b) Find the distance of the centre of mass of the framework from *A*. **(3)**

(a)

	AC	FD	BE	CD	framework
Mass	6a	6a	4a	4a	24a
Distance	3a	3a	4a	6a	d

Moments about AF: (1) $6a \times 3a + 6a \times 3a + 4a \times 4a + 4a \times 6a = 24a \times d$ (2)

$76a^2 = 24ad$

$d = \dfrac{19a}{6}$ (1)

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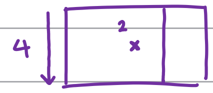
Question 1 continued

(b) $\bar{y} = \frac{4a}{2} = 2a$ (1) ← uniform, so c.o.m is half way 'down' the framework.



$D^2 = \left(\frac{19a}{6}\right)^2 + (2a)^2$

$D^2 = \frac{316a^2}{36} + 4a^2$ (1)

$D = \sqrt{\frac{505}{36}a^2}$

$D = \frac{\sqrt{505}}{6}a$ (1)

(Total for Question 1 is 7 marks)

2.

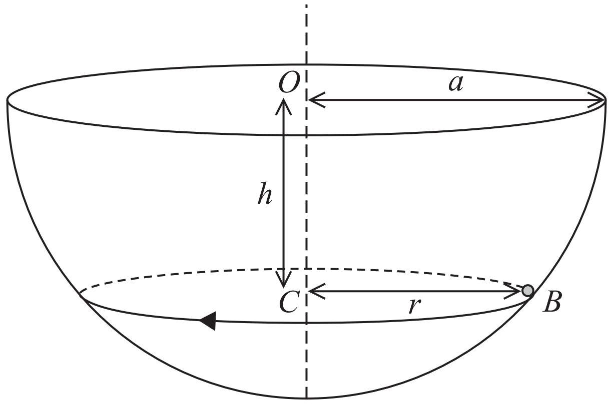


Figure 2

A thin hollow hemisphere, with centre O and radius a, is fixed with its axis vertical, as shown in Figure 2.

A small ball B of mass m moves in a horizontal circle on the inner surface of the hemisphere. The circle has centre C and radius r. The point C is vertically below O such that $OC = h$.

The ball moves with constant angular speed ω

The inner surface of the hemisphere is modelled as being smooth and B is modelled as a particle. Air resistance is modelled as being negligible.

(a) Show that $\omega^2 = \dfrac{g}{h}$

(6)

Given that the magnitude of the normal reaction between B and the surface of the hemisphere is $3mg$

(b) find ω in terms of g and a.

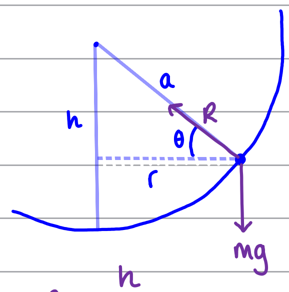
(3)

(c) State how, apart from ignoring air resistance, you have used the fact that B is modelled as a particle.

(1)

(a) Resolve horizontally $F = m \times a$ ①

$R\cos\theta = m \times r\omega^2$ ① ①

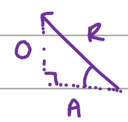


Resolve vertically: ① $mg = R\sin\theta$ ② ①

$\tan\theta = \frac{h}{r}$

② − ① : $\dfrac{mg}{mr\omega^2} = \dfrac{R\sin\theta}{R\cos\theta}$

$\dfrac{g}{r\omega^2} = \tan\theta$



↑ $\sin\phi = \frac{O}{R}$

← $\cos\phi = \frac{A}{R}$

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Question 2 continued

from diagram → $\tan\theta = \frac{h}{r}$ ⟹ $\frac{g}{r\omega^2} = \frac{h}{r}$ (1)

$\times r$

$\frac{rg}{r\omega^2} = h$

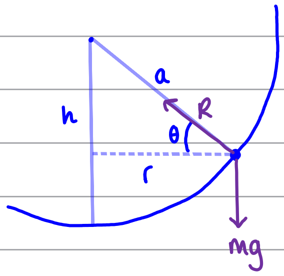
$\times \omega^2$

$g = h\omega^2$

$\div h$

$\frac{g}{h} = \omega^2$ (1)

(b)



Resolve horizontally: $F = m \times a$

$mr\omega^2 = 3mg\cos\theta$ (1)

$\cos\theta = \frac{r}{a}$ ⟹ $r = a\cos\theta$

$ma\cos\theta\omega^2 = 3mg\cos\theta$

$\div m\cos\theta$

$a\omega^2 = 3g$ (1)

$\div a$

$\omega^2 = \frac{3g}{a}$

$\omega = \sqrt{\frac{3g}{a}}$ (1)

(c) Have ignored the dimensions of B by modelling it as a particle (1)

Question 2 continued

Question 2 continued

(Total for Question 2 is 10 marks)

3. A particle P is moving along the x-axis. At time t seconds, P has velocity $v\,\text{m s}^{-1}$ in the positive x direction and acceleration $a\,\text{m s}^{-2}$ in the positive x direction.

In a model of the motion of P

$$a = 4 - 3v$$

When $t = 0$, $v = 0$

(a) Use integration to show that $v = k(1 - e^{-3t})$, where k is a constant to be found.

(7)

When $t = 0$, P is at the origin O

(b) Find, in terms of t only, the distance of P from O at time t seconds.

(4)

(a) $a = 4 - 3v$

$\frac{dv}{dt} = 4 - 3v$ $a = \frac{dv}{dt}$

$\times dt$, $\div (4-3v)$

$\frac{1}{4-3v}\,dv = 1\,dt$ ①

$\int \frac{1}{4-3v}\,dv = \int 1\,dt$ ①

$\int \frac{1}{x}\,dx = \ln|x|$

$-\frac{1}{3}\ln|4-3v| = t + c$ ①

When $t = 0$, $v = 0$:

$-\frac{1}{3}\ln|4 - 3(0)| = 0 + c$ ①

$-\frac{1}{3}\ln 4 = c$

$-\frac{1}{3}\ln|4-3v| = t - \frac{1}{3}\ln 4$

$t = \frac{1}{3}\ln 4 - \frac{1}{3}\ln|4-3v|$

$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$

$t = \frac{1}{3}\ln\left(\frac{4}{4-3v}\right)$ ①

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Question 3 continued

$$e^{3t} = \frac{4}{4-3v} \quad (1)$$

$\frac{1}{e^{3t}} = e^{-3t}$

$$4 - 3v = 4e^{-3t}$$

$$4 - 4e^{-3t} = 3v$$

$$\frac{4}{3}(1 - e^{-3t}) = v \quad (1)$$

(b)

$$v = k(1 - e^{-3t})$$

$v = \frac{dx}{dt}$ to find position x

$$\frac{dx}{dt} = k(1 - e^{-3t})$$

split and integrate both sides

$$\int 1 \, dx = \int k(1 - e^{-3t}) \, dt \quad (1)$$

$$x = k\left(t - \left(-\tfrac{1}{3}\right)e^{-3t}\right) + c$$

$k = \frac{4}{3}$ from (a)

$$x = \frac{4}{3}\left(t + \frac{1}{3}e^{-3t}\right) + c \quad (1)$$

When $t = 0$, $x = 0$: ← P is at the origin O, so $x = 0$.

$$0 = \frac{4}{3}\left(0 + \frac{1}{3}e^{-3 \times 0}\right) + c \quad (1)$$

$$0 = \frac{4}{3}\left(\frac{1}{3}\right) + c$$

$$c = -\frac{4}{9}$$

$$x = \frac{4}{3}\left(t + \frac{1}{3}e^{-3t}\right) - \frac{4}{9}$$

$$x = \frac{4}{3}\left(t + \frac{1}{3}e^{-3t} - \frac{1}{3}\right) \quad (1)$$

Question 3 continued

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Question 3 continued

(Total for Question 3 is 11 marks)

4.

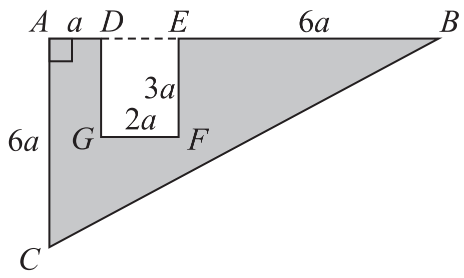

Figure 3

The uniform triangular lamina ABC has AB perpendicular to AC, $AB = 9a$ and $AC = 6a$. The point D on AB is such that $AD = a$.

The rectangle $DEFG$, with $DE = 2a$ and $EF = 3a$, is removed from the lamina to form the template shown shaded in Figure 3.

The distance of the centre of mass of the template from AC is d.

(a) Show that $d = \frac{23}{7}a$

(3)

The template is freely suspended from A and hangs in equilibrium with AB at an angle $\theta°$ to the downward vertical through A.

(b) Find the value of θ

(5)

A new piece, of exactly the same size and shape as the template, is cut from a lamina of a different uniform material. The template and the new piece are joined together to form the model shown in Figure 4. Both parts of the model lie in the same plane.

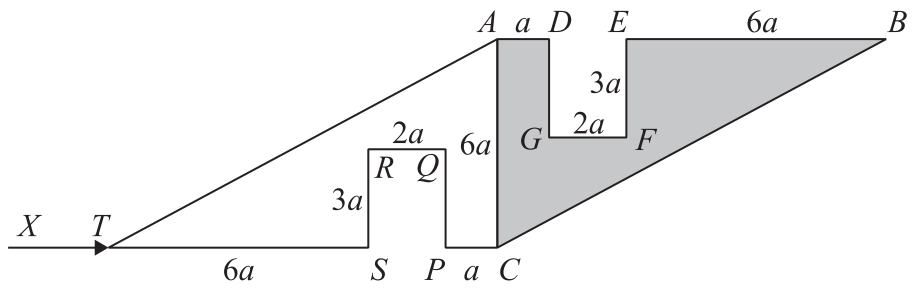

Figure 4

The weight of $CPQRSTA$ is W

The weight of $ADGFEBC$ is $4W$

The model is freely suspended from A.

A horizontal force of magnitude X, acting in the same vertical plane as the model, is now applied to the model at T so that AC is vertical, as shown in Figure 4.

(c) Find X in terms of W.

(4)

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Question 4 continued

(a)

	ABC	DEFG	lamina
Mass	$27a^2$	$6a^2$	$(27-6)a^2$
Distance	$3a$	$2a$	d

Moments about AC : $27a^2 \times 3a - 6a^2 \times 2a = 21a^2d$ (1) (1)

the rectangle has been removed

$69a^3 = 21a^2d$

$\frac{69a^3}{21a^2} = d$

$\frac{23a}{7} = d$ (1)

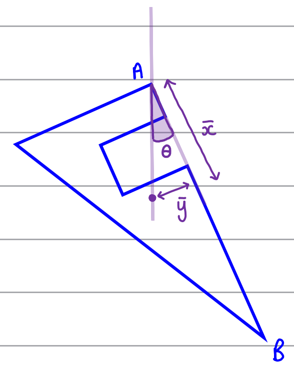
(b) Moments about AB (1)

$(27-6)a^2\bar{y} = 27a^2 \times 2a - 6a^2 \times 1.5a$ (1)

$21a^2\bar{y} = 45a^3$

$\bar{y} = \frac{45a}{21}$ (1)

$\tan\theta = \frac{\bar{y}}{\bar{x}}$ (1) ← $\tan\theta = \frac{O}{A}$



$\theta = \tan^{-1}\left(\frac{\frac{23a}{7}}{\frac{45a}{21}}\right)$

$\theta = \tan^{-1}\left(\frac{15}{23}\right)$

$\theta = 33.11°$ (1)

Question 4 continued

(c) Moments about A : ①

$$6a \times X + \frac{23}{7}a \times W = \frac{23}{7}a \times 4W$$ ①

↑ force X at T ↑ left-hand lamina ↑ right-hand lamina

$$6X + \frac{23}{7}W = \frac{92}{7}W$$

$$6X = \frac{69}{7}W$$

$$X = \frac{23}{14}W$$ ①

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Question 4 continued

Question 4 continued

(Total for Question 4 is 12 marks)

TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS